Roll No. | | | | | | | | | | |

## ANNA UNIVERSITY (UNIVERSITY DEPARTMENTS)

**B.E / B. Tech (Full Time) END SEMESTER EXAMINATIONS – APRIL / MAY 2019**

COMMON TO ALL BRANCH

Semester iii

**MA7302 – Partial Differential Equation**

(Regulation 2015 – Mathematics)

Time: 3 Hours | Answer ALL Questions | Max. Marks 100

### PART- A (10 x 2 = 20 Marks)

| Q.No | Questions | Marks |
|---|---|---|
| 1. | Eliminate the arbitrary constants a & b from $z = (x^2+a)(y^2+b)$ | 2 |
| 2. | Solve $(D^2 - 4DD' + 3D'^2)z = 0$ | 2 |
| 3. | Define Dirichlet's Conditions. | 2 |
| 4. | Write the formula of Fourier constants to expand f(x) in $(-l, l)$. | 2 |
| 5. | A string is stretched and fastened to two points $l$ apart. Motion is started by displacing the string into the form $y = y_0 sin\frac{\pi x}{l}$ from which it is released at time t = 0. Formulate this problem as the boundary value problem. | 2 |
| 6. | Write down the three possible solutions of Laplace equation in two-dimensions. | 2 |
| 7. | Define Thomas algorithm. | 2 |
| 8. | Define Alternating Direction implicit (ADI) method | 2 |
| 9. | Distinguish between direct and iterative method of solving simultaneous equation. | 2 |
| 10. | What is the purpose of Liebmann's process? | 2 |

### PART- B (5 x 16 = 80 Marks)

(Q. No 11 is Compulsory)

| Q.No | Questions | Marks |
|---|---|---|
| 11. | A rod 30 cm long has its ends A and B kept at 20° and 80° respectively until steady state conditions prevail. The temperature at each end is then suddenly reduced to 0°C and kept so. Find the resulting temperature function $u(x,t)$ taking x = 0 at A. | 16 |
| 12. | a) (i) Solve: $p^2 + x^2y^2q^2 = x^2z^2$ | 8 |
| | (ii) Solve: $z^2(p^2+q^2) = x^2+y^2$ | 8 |
| | OR | |
| | b) (i) Solve $(y+z)p + (z+x)q = x+y$ | 8 |
| | (ii) Solve $(D^2 + 2DD' + D'^2)z = x^2y + e^{x-y}$ | 8 |
| 13. | a)(i) If $f(x) = \frac{1}{2}(\pi - x)$, find the Fourier series for period 2π in the interval (0, 2π). Hence deduce that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \frac{\pi}{4}$ | 10 |
| | (ii) Find the Half range Fourier cosine series of f(x) = $(x-1)^2$ in $(0,1)$. | 6 |
| | OR | |
| | b)(i) Determine the first two harmonic of the Fourier series for the following values. | 10 |

| X: | 0 | $\frac{\pi}{3}$ | $\frac{2\pi}{3}$ | $\pi$ | $\frac{4\pi}{3}$ | $\frac{5\pi}{3}$ |
|---|---|---|---|---|---|---|
| Y: | 1.98 | 1.30 | 1.05 | 1.30 | -0.88 | -0.25 |

| | | |
|---|---|---|
| | (ii) Find the Half range Fourier sine series of f(x) = x in $(0, l)$. | **6** |
| 14. | a) Solve the following system of equations by Gaussian elimination methods<br>$x + y + z = 9$<br>$2x - 3y + 4z = 13$<br>$3x + 4y + 5z = 40$ | **16** |
| | **OR** | |
| | b) Solve by Crank-Nicolson's method $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$ for 0 < x < 1, t > 0, given that $u(0,t) = 0, u(1,t) = 0 \; and \; u(x,0) = 100(x - x^2)$. | **16** |
| 15. | a) Solve the following system of equations using Gauss Seidel iteration method.<br>$10x + 2y + z = 9$<br>$x + 10y - z = -22$<br>$-2x + 3y + 10z = 22$ | **16** |
| | **OR** | |
| | b) Solve $\nabla^2 u = -10(x^2 + y^2 + 10)$ over the square with sides x = 0 = y; x = 3 = y; with u = 0 on the boundaries and mesh length 1. | **16** |